

New examples of the representation of 1 by the sum of reciprocals of semiprime numbers

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Abstract

In 1978, Allan.Wm.Johnson obtained an example of the representation of 1 by the sum of reciprocals of the product of two distinct prime numbers. His example has 48 terms [1], and we had no examples which have less than 48 terms until now. In this paper, we construct 17 new examples that have less than 48 terms. Since all of the new examples have 47 terms and we have no examples which have less than 47 terms, it is assumed that the minimum number of terms is 47.

1 Introduction

In this note, we consider the representation of 1 by the finite sum of some reciprocals where each denominator is the product of two distinct primes. These representations of 1 are equivalent to solutions of a Diophantine equation below:

$$\sum_{i=1}^n \frac{1}{x_i} = 1, \quad \text{where } 2 \times 3 \leq x_1 < x_2 < \cdots < x_n \quad (1)$$

when $x_i = p_i q_i$, $p_i < q_i$ are primes for each i , n is the number of terms.

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1.1 Previous research and our problems

In 1977, Barbeau obtained a solution of (1) which has 101 terms ($n = 101$)[2]. In 1978, Allan.Wm.Johnson improved Barbeau's result and exhibited a solution of (1) which has 48 terms ($n = 48$)[1].

Richard K.Guy questioned whether the minimum number of terms is 48 [3]. Here is Johnson's example:

$$\begin{aligned} 1 = & \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} \\ & + \frac{1}{39} + \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} \\ & + \frac{1}{85} + \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{133} \\ & + \frac{1}{155} + \frac{1}{187} + \frac{1}{203} + \frac{1}{209} + \frac{1}{215} + \frac{1}{221} + \frac{1}{247} + \frac{1}{265} + \frac{1}{287} \\ & + \frac{1}{299} + \frac{1}{319} + \frac{1}{323} + \frac{1}{391} + \frac{1}{689} + \frac{1}{731} + \frac{1}{901} \end{aligned}$$

No examples which have 47 or fewer terms are discovered. In 2015, Nechemia Burshtein conjectured that there are other examples of (1) which have 48 terms [4].

Our problem is the following two;

- To construct the new examples of (1) which has 47 or less terms. ($n \leq 47$)
- To consider the minimum number of terms.

2 Main results

We have the new 17 examples of (1) which have 47 terms.

3 Constructions

3.1 Reconsideration of Johnson's example

We factorize all denominators of Johnson's example.

$6 = 2 \times 3,$	$10 = 2 \times 5,$	$14 = 2 \times 7,$	$15 = 3 \times 5,$	$21 = 3 \times 7,$	$22 = 2 \times 11,$
$26 = 2 \times 13,$	$33 = 3 \times 11,$	$34 = 2 \times 17,$	$35 = 5 \times 7,$	$38 = 2 \times 19,$	$39 = 3 \times 13,$
$46 = 2 \times 23,$	$51 = 3 \times 17,$	$55 = 5 \times 11,$	$57 = 3 \times 19,$	$58 = 2 \times 29,$	$62 = 2 \times 31,$
$65 = 5 \times 13,$	$69 = 3 \times 23,$	$77 = 7 \times 11,$	$82 = 2 \times 41,$	$85 = 5 \times 17,$	$86 = 2 \times 43,$
$87 = 3 \times 29,$	$91 = 7 \times 13,$	$93 = 3 \times 31,$	$95 = 5 \times 19,$	$115 = 5 \times 23,$	$119 = 7 \times 17,$
$123 = 3 \times 41,$	$133 = 7 \times 19,$	$155 = 5 \times 31,$	$187 = 11 \times 17,$	$203 = 7 \times 29,$	$209 = 11 \times 19,$
$215 = 5 \times 43,$	$221 = 13 \times 17,$	$247 = 13 \times 19,$	$265 = 5 \times 53,$	$287 = 7 \times 41,$	$299 = 13 \times 23,$
$319 = 11 \times 29,$	$323 = 17 \times 19,$	$391 = 17 \times 23,$	$689 = 13 \times 53,$	$731 = 17 \times 43,$	$901 = 17 \times 53.$

We call the product of two distinct primes as *a semiprime number*.

Since semiprime numbers exist infinitely, it is impossible to calculate all patterns under no restriction on denominators.

In the above table, all of the prime factors are less than 53. So in this study, we restrict that the maximum of the prime factors of denominators is 53.

There are 120 semiprimes obtained by multiplying different two prime numbers from 2 to 53.

3.2 Constructions of examples of 47 terms

We fix the vector \mathbf{p}_f in 120 dimensional vector space as follows:

$$\mathbf{p}_f = \left(\frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{15}, \dots, \frac{1}{2491} \right) \in \mathbb{R}^{120}$$

This is a 120-dimensional vector having the reciprocals of semiprime numbers in descending order as components. For \mathbf{p}_f , we correspond to the following another vector $\mathbf{v} \in \mathbb{R}^{120}$:

$$\mathbf{v} = (a_1, a_2, \dots, a_{120}) \in \mathbb{R}^{120}$$

Each a_i takes a value of only 0 or 1. It takes 1 if the component of \mathbf{p}_f appears in solutions of (1), otherwise it takes 0. For example, if $\frac{1}{6}$ which is the first component of \mathbf{p}_f appears in the representation, the first component a_1 of \mathbf{v} is equal to 1. The number of reciprocals of semiprime numbers appearing in the solution of (1) is equal to the following inner product:

$$A = \mathbf{v} \cdot {}^t(1, 1, \dots, 1).$$

Where $(1, 1, \dots, 1) \in \mathbb{R}^{120}$, all components of this vector are 1. Of course, we aim for $A = 47$, but it is difficult, we decide in part the components of

\mathbf{v} . Let $\mathbf{v}(n)$ be the vector determined up to the n -th component of \mathbf{v} and all other components are 0. Consider the other inner product,

$$B(n) = \mathbf{v}(n) \cdot {}^t \mathbf{p}_f.$$

$B(n)$ is a summation up to n -th terms. We make 1 by adding $B(n)$ and some reciprocals of semiprime numbers. We make a tree diagram for adding semiprime numbers or not, one by one to B , starting from the n -th smallest semiprime number according to the following proposition 1. These observations are summarized in the following two propositions. And we consider the fixed vector $\mathbf{q}_f \in \mathbb{R}^{120}$ where $\mathbf{q}_f = (6, 10, 14, 15, \dots, 2491)$. All of the components of \mathbf{q}_f are semiprime numbers which are the products of two different primes, its bigger prime factor is up to 53. We assume that each component of \mathbf{q}_f is arranged in ascending order. And each component of \mathbf{q}_f is also reciprocal of corresponding component of \mathbf{p}_f .

Proposition 1 *We assume that q_i represents the i -th component of \mathbf{q}_f , $6 = q_1, 10 = q_2, 14 = q_3, \dots, 2491 = q_{120}$.*

1.1 *When the components of \mathbf{v} are determined from a_1 to a_n , the inequality*

$$B(n) + \sum_{k=1}^{47-C(n)} \frac{1}{q_{n+k}} < 1 \text{ does not hold, where } C(n) = \mathbf{v}(n) \cdot {}^t (1, 1, \dots, 1).$$

1.2 *For any prime numbers $r < 53$, when the components of \mathbf{v} are determined from a_1 to a_n , and the corresponding n -th component of the vector \mathbf{q}_f is greater than $53r$, multiples of r do not appear in the subsequent semiprime numbers. Therefore, the common denominator of the partial sum up to the above condition must not be a multiple of r .*

We found 2 new examples of (1) consisting of 47 terms, and also we found 94 examples consisting of 48 terms with this restriction, including Johnson's example.

3.3 Solutions of (1) which have 47 terms

Here are our new two examples of 47 terms.

$$\begin{aligned}
 1 = & \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
 & + \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{74} + \frac{1}{82} + \frac{1}{85} + \frac{1}{86} \\
 & + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{106} + \frac{1}{111} + \frac{1}{123} + \frac{1}{133} + \frac{1}{145} + \frac{1}{155} + \frac{1}{159} \\
 & + \frac{1}{185} + \frac{1}{203} + \frac{1}{215} + \frac{1}{253} + \frac{1}{265} + \frac{1}{287} + \frac{1}{319} + \frac{1}{493} + \frac{1}{583} \\
 & + \frac{1}{731} + \frac{1}{851} + \frac{1}{1073}
 \end{aligned}$$

$$\begin{aligned}
 1 = & \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
 & + \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{74} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} + \frac{1}{87} \\
 & + \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} + \frac{1}{143} \\
 & + \frac{1}{145} + \frac{1}{155} + \frac{1}{161} + \frac{1}{221} + \frac{1}{253} + \frac{1}{259} + \frac{1}{287} + \frac{1}{299} + \frac{1}{391} \\
 & + \frac{1}{473} + \frac{1}{481} + \frac{1}{1247} + \frac{1}{1591}
 \end{aligned}$$

Later, we shifted the upper bound of prime factors of denominators from 53 to 101. The similar consideration and calculation in a 325-dimensional vector space, with some minor revisions, we obtained more 15 examples consisting of 47 terms. But there were no examples which have 46 or fewer terms under this restriction.

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{74} + \frac{1}{77} + \frac{1}{86} \\
&+ \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{118} + \frac{1}{119} + \frac{1}{129} + \frac{1}{133} \\
&+ \frac{1}{145} + \frac{1}{155} + \frac{1}{161} + \frac{1}{185} + \frac{1}{187} + \frac{1}{329} + \frac{1}{517} + \frac{1}{667} + \frac{1}{851} + \frac{1}{1073} \\
&+ \frac{1}{1357} + \frac{1}{1363} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{133} + \frac{1}{145} \\
&+ \frac{1}{155} + \frac{1}{161} + \frac{1}{187} + \frac{1}{203} + \frac{1}{215} + \frac{1}{287} + \frac{1}{329} + \frac{1}{493} + \frac{1}{517} + \frac{1}{1247} \\
&+ \frac{1}{1357} + \frac{1}{1363} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{115} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} \\
&+ \frac{1}{141} + \frac{1}{143} + \frac{1}{145} + \frac{1}{155} + \frac{1}{287} + \frac{1}{377} + \frac{1}{391} + \frac{1}{551} + \frac{1}{799} + \frac{1}{893} \\
&+ \frac{1}{1357} + \frac{1}{1363} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{115} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} \\
&+ \frac{1}{143} + \frac{1}{145} + \frac{1}{155} + \frac{1}{177} + \frac{1}{287} + \frac{1}{299} + \frac{1}{391} + \frac{1}{551} + \frac{1}{611} + \frac{1}{799} \\
&+ \frac{1}{893} + \frac{1}{1711} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{86} \\
&+ \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} + \frac{1}{145} \\
&+ \frac{1}{155} + \frac{1}{161} + \frac{1}{177} + \frac{1}{187} + \frac{1}{203} + \frac{1}{287} + \frac{1}{299} + \frac{1}{329} + \frac{1}{377} + \frac{1}{517} \\
&+ \frac{1}{1363} + \frac{1}{1711} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{74} + \frac{1}{77} + \frac{1}{82} \\
&+ \frac{1}{86} + \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{111} + \frac{1}{119} + \frac{1}{122} + \frac{1}{123} + \frac{1}{133} + \frac{1}{143} + \frac{1}{145} \\
&+ \frac{1}{155} + \frac{1}{161} + \frac{1}{183} + \frac{1}{187} + \frac{1}{259} + \frac{1}{287} + \frac{1}{299} + \frac{1}{319} + \frac{1}{473} + \frac{1}{481} \\
&+ \frac{1}{559} + \frac{1}{671} + \frac{1}{1591}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} \\
&+ \frac{1}{155} + \frac{1}{161} + \frac{1}{187} + \frac{1}{213} + \frac{1}{287} + \frac{1}{329} + \frac{1}{355} + \frac{1}{493} + \frac{1}{497} + \frac{1}{517} \\
&+ \frac{1}{1357} + \frac{1}{1363} + \frac{1}{2537}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{74} + \frac{1}{77} + \frac{1}{82} \\
&+ \frac{1}{85} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} + \frac{1}{143} \\
&+ \frac{1}{145} + \frac{1}{155} + \frac{1}{187} + \frac{1}{213} + \frac{1}{221} + \frac{1}{287} + \frac{1}{301} + \frac{1}{559} + \frac{1}{629} + \frac{1}{781} \\
&+ \frac{1}{1247} + \frac{1}{1591} + \frac{1}{1633}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{133} + \frac{1}{141} + \frac{1}{142} \\
&+ \frac{1}{143} + \frac{1}{155} + \frac{1}{203} + \frac{1}{221} + \frac{1}{235} + \frac{1}{287} + \frac{1}{299} + \frac{1}{355} + \frac{1}{377} + \frac{1}{391} \\
&+ \frac{1}{559} + \frac{1}{2021} + \frac{1}{2059}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{119} + \frac{1}{133} + \frac{1}{141} \\
&+ \frac{1}{142} + \frac{1}{155} + \frac{1}{187} + \frac{1}{205} + \frac{1}{213} + \frac{1}{235} + \frac{1}{493} + \frac{1}{517} + \frac{1}{1363} + \frac{1}{1633} \\
&+ \frac{1}{1763} + \frac{1}{1927} + \frac{1}{2627}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{86} \\
&+ \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{133} + \frac{1}{141} + \frac{1}{142} \\
&+ \frac{1}{143} + \frac{1}{145} + \frac{1}{155} + \frac{1}{203} + \frac{1}{213} + \frac{1}{221} + \frac{1}{235} + \frac{1}{287} + \frac{1}{493} + \frac{1}{559} \\
&+ \frac{1}{1633} + \frac{1}{2021} + \frac{1}{2627}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{74} + \frac{1}{77} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{119} + \frac{1}{133} + \frac{1}{141} \\
&+ \frac{1}{142} + \frac{1}{145} + \frac{1}{155} + \frac{1}{187} + \frac{1}{213} + \frac{1}{517} + \frac{1}{629} + \frac{1}{799} + \frac{1}{1247} + \frac{1}{1591} \\
&+ \frac{1}{1633} + \frac{1}{2021} + \frac{1}{2627}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{111} + \frac{1}{115} + \frac{1}{118} + \frac{1}{119} + \frac{1}{123} + \frac{1}{133} \\
&+ \frac{1}{142} + \frac{1}{155} + \frac{1}{187} + \frac{1}{213} + \frac{1}{287} + \frac{1}{295} + \frac{1}{329} + \frac{1}{413} + \frac{1}{493} + \frac{1}{517} \\
&+ \frac{1}{1363} + \frac{1}{1633} + \frac{1}{2627}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{87} \\
&+ \frac{1}{91} + \frac{1}{93} + \frac{1}{95} + \frac{1}{106} + \frac{1}{111} + \frac{1}{115} + \frac{1}{119} + \frac{1}{122} + \frac{1}{123} + \frac{1}{133} + \frac{1}{142} \\
&+ \frac{1}{155} + \frac{1}{159} + \frac{1}{183} + \frac{1}{187} + \frac{1}{203} + \frac{1}{213} + \frac{1}{265} + \frac{1}{287} + \frac{1}{319} + \frac{1}{583} \\
&+ \frac{1}{671} + \frac{1}{1633} + \frac{1}{2627}
\end{aligned}$$

$$\begin{aligned}
1 &= \frac{1}{6} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{21} + \frac{1}{22} + \frac{1}{26} + \frac{1}{33} + \frac{1}{34} + \frac{1}{35} + \frac{1}{38} + \frac{1}{39} \\
&+ \frac{1}{46} + \frac{1}{51} + \frac{1}{55} + \frac{1}{57} + \frac{1}{58} + \frac{1}{62} + \frac{1}{65} + \frac{1}{69} + \frac{1}{77} + \frac{1}{82} + \frac{1}{85} \\
&+ \frac{1}{86} + \frac{1}{87} + \frac{1}{91} + \frac{1}{93} + \frac{1}{94} + \frac{1}{95} + \frac{1}{115} + \frac{1}{119} + \frac{1}{123} + \frac{1}{129} + \frac{1}{133} \\
&+ \frac{1}{141} + \frac{1}{142} + \frac{1}{145} + \frac{1}{155} + \frac{1}{187} + \frac{1}{517} + \frac{1}{799} + \frac{1}{1207} + \frac{1}{1247} + \frac{1}{1633} \\
&+ \frac{1}{1763} + \frac{1}{2021} + \frac{1}{2911}
\end{aligned}$$

4 Consideration of the minimum number of terms and future works

4.1 No 46-term example

We proved that there are no 46-term examples when the maximum prime factor is 101. Also, the maximum prime factors of denominators of our 17 examples which have 47 terms, are 71, so there is a low possibility of the existence of another example having 47 terms except for our 17 examples.

4.2 Future works

Our future works are to expand the dimension of vector space and proof that the minimum number of terms is 47.

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References

- [1] Allan.WM.Johnson,*Letter to editor*, Crux Mathematicorum (=Eureka (Ottawa)), 4 (1978) 190.
- [2] E.J.Barbeau, *Expressing one as a sum of distinct reciprocals* : Comments and a bibliography,Eureka (Ottawa) 3 (1977) p178-p181
- [3] Richard K. Guy,*Unsolved Problems in Number Theory*
- [4] Nechemia Burshtein, *On distinct unit fractions whose sum equals 1 when $x_i \nmid x_j$ for $i \neq j$* , Journal for Algebra and Number Theory Academia Volume 5, issue 4, Dec.2015, p117-p124.