

Let's use the symbol  $S_n$  to represent a string of  $n$  S's. So, for example,  $S_7 = \text{SSSSSSS}$ .

In Leyland's formula he defines a string of  $n$  S's to mean "start with the number 2 and take the square root  $n$  times", so

$$S_n = 2^{\frac{1}{2^n}}$$

and thus

$$\lg(S_n) = \lg\left(2^{\frac{1}{2^n}}\right) = \frac{1}{2^n} = 2^{-n}$$

where "lg" means "log base 2". Therefore,

$$\lg(\lg(S_n)) = \lg(2^{-n}) = -n \quad (1)$$

Leyland's formula for  $\pi$  is

$$\frac{\lg(\lg(S_{22}))}{\lg(\lg(S_7))}$$

so by equation (1) this equals  $-22 / -7 = 22/7 = 3.142857\dots$