

This conjecture must be considered in the context of the Riemann Zeta-function, which exists in its real part since Euler. It exists in the form of an infinite sum by using the natural numbers and in the form of an infinite product using the primes. Since π is represented as an infinite series or as an infinite product, one can already see a possible connection between π and the prime numbers. This is confirmed by the relations $\text{Zeta}(2) = \pi^2/6$, $\text{Zeta}(4) = (\pi^4)/90$, generally $\text{Zeta}(2k) = (\pi^{2k})/r$, where r is rational. $\text{Zeta}(1)$ yields the harmonic series that is known to diverge. $\text{Zeta}(1)$ in product form is $= \text{Product}(p/(p-1)) = \text{Product}(1/(1-1/p))$ over all primes p . Since $\text{Zeta}(1) = \text{infinity}$, there is not much to do with it. Admitting, however, only the twin primes instead of the primes, things look different: the product does not seem to get beyond π : this is the actual conjecture. Talking about twin primes here always means the minor partner of the twins. The conjecture works as well with the major partner, but the convergence is much weaker. Admitting both partners, π^2 would be expected as a result, but the convergence is of course bad, too. In order to make clear that, in contrast to the primes, only the twins are processed, it seems sensible to write t instead of p : $\text{Product}(t/(t-1)) = \pi$.

If we set $u = t-1$, the product is obtained in the form of $\text{Product}(1 + 1/u)$ ($u = 2 \dots \text{inf}$). We can recognize that the factor $(1 + 1/u)$ goes from top to 1, so the product increases monotonously. Since the series $\text{Sum}(1/u)$ converges ($<$ Brun's constant), the $\text{Product}(1 + 1/u) = \text{Product}(1/(1-1/t))$ is convergent (convergence criterion for infinite products).

There is an interesting relationship to the **Wallis Product** $\text{Product}[(2k)^2 / ((2k)^2 - 1)] = \pi / 2$. Substituting the factor 2 by $e/2$, the result is probably π . This is another **conjecture**.

On the whole, the statement $\text{Product}(t/(t-1)) = \pi$ remains a conjecture.

My most recent values are: $t = 30\,048\,993\,899$, $\text{Product}(t) = 3.02596438336974491136$.
Without warranty, since rounding errors may occur.

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